# Pion (Kaon) and Sigma polarizabilities [♯]

S. Bellucci

INFN-Laboratori Nazionali di Frascati,
P.O.Box 13, 00044 Frascati, Italy

## Abstract

We report the results of the working group on "Pion (Kaon) and Sigma Polarizabilities". Interesting possibilities to measure these polarizabilities include the radiative pion photoproduction in the MAMI experiment at Mainz, as well as at the GRAAL facility (actually the latter is being considered for an experimental determination of the pion polarizabilities here for the first time), the experimental plans on Primakoff effect at FNAL, and the measurements at the Frascati Φ-factory DAΦNE.

# 1 Introduction

We report here about the activity of the working group on "Pion (Kaon) and Sigma Polarizabilities" we conducted, in collaboration with R. Baldini, at the Workshop on Chiral Dynamics: Theory and Experiment, held July 25-29,1994 at MIT. The goal of this working group was to identify the processes that are more suitable to measure the electric and magnetic polarizabilities of the abovementioned hadrons and probe chiral dynamics in the photon-pion (-kaon) and photon-sigma physics.

The agenda of the group was a mixture of theory and experiment that allowed us to summarize the current status in this field, determine what is to be done in order to improve it, from both the theoretical and the experimental side, and quantify the level of accuracy needed to make the improvement significant.

We considered the following general areas:

1. Theoretical predictions and models for Compton scattering $\gamma\pi(K) \to \gamma\pi(K)$, as well as for $\gamma\gamma \to \pi\pi(KK)$, and the relation to the pion (kaon) polarizabilities. 2. Experiments to measure the pion (kaon) and sigma polarizabilities. 3. Phenomenology required in order to extract the polarizabilities from the experimental data.

We have considered the following methods of measurement:

i) Radiative photoproduction of the pion and extrapolating to the pion pole, in order to extract the polarizabilities form the data. ii) Experiments to measure the pseudoscalar meson polarizabilities using the Primakoff effect. iii) The measurements at the Frascati DA$\Phi$NE with the KLOE detector.

Murray Moinester illustrated the plans at FNAL and the reaction $\pi\rho \to \pi\gamma$ in connection with the pion and sigma polarizabilities. Thomas Walcher went over the MAMI project at Mainz. Annalisa D'Angelo discussed the potential of the Graal synchrotron light facility in Grenoble to carry out polarizability measurements. S. Kananov discussed the need for a careful estimate of the radiative corrections for the FNAL experiments where the pion polarizabilities can be measured. A presentation of the DA$\Phi$NE capabilities has been done in the workshop.

The related talks had the following titles: A. D'Angelo: "The experimental plans in Grenoble", T. Walcher: "The MAMI experiment at Mainz", M. Moinester: "Pion polarizabilities and quark-gluon plasma signatures", S. Kananov: "Radiative corrections for pion polarizability experiments",

In the second part of the working group, devoted to the theoretical contributions, we had the following talks: J. Kambor: "Determination of a $O(p^6)$ counterterm from sum rules", M. Knecht: "$\gamma\gamma \to \pi^0\pi^0$ and $\pi^0$ polarizabilities in generalized chiral perturbation theory", M. Pennington: "Dispersion relations and pion polarizabilities", S. Bellucci: "Difficulties in extrapolating to the pion pole the data on radiative pion photoproduction".

# 2 Experimental plans for polarizability measurements

## 2.1 Radiative pion photoproduction

Let us consider first of all the unexpected and very innovative contribution by Annalisa D'Angelo. She has reported about the possibility to measure the pion polarizability by Graal, the new facility at the electron storage ring ESRF in Grenoble.

The Graal facility consists of a tagged and highly polarized $\gamma$-ray beam, produced by the backscattering of Laser light against the high energy electrons circulating in the storage ring ESRF at Grenoble [1]-[3].

If commercial Ion-Argon and Nd-Yag Lasers are used, either linearly or circularly polarized, a $\gamma$-ray beam of energy ranging from about 300 MeV to 1.5 GeV with a degree of polarization higher then 70% over almost the entire spectrum may be obtained, by appropriate choice of the Laser line.

A large solid angle multi purpose detector [4] is part of the Graal facility and it will be used to perform experiments on photo-nuclear reactions [5]. It consists of a crystal ball made of 480 BGO crystals (24 cm long) covering all azimuthal angles for polar angles in the interval between 25° and 155°. It may be used as electromagnetic calorimeter, with measured energy resolution of 2% FWHM for 1 GeV photons, or to detect protons of energy up to 300 MeV [6]-[8].

The central hole of the BGO ball ($\phi = 20$ cm) will contain a barrel of 32 plastic scintillators; it will be used to discriminate between charged and neutral particles and to identify the charged particles with the $\frac{\Delta E}{E}$ technique. Inside the barrel two cylindrical wire chambers will provide improved angular resolution for the reconstruction of the trajectories of the charged particles.

In the forward direction two detectors, each consisting of three plane wire cham-

bers rotated of 45°, and a scintillating wall will provide charged particle tracking information and TOF measurements; 10% efficiency is expected for neutron detection in the scintillating wall.

We have started to investigate the possibility of using the Graal facility to study the radiative pion photoproduction from the proton (namely the reaction $\vec{\gamma} + p \rightarrow n + \pi^+ + \gamma$), in order to extrapolate the experimental data to the pion pole and determine the cross-section of the Compton scattering on the pion [9].

The interest of this measure, in order to get information on the pion polarizability, has been pointed out, among other authors, by D. Drechsel and L.V. Fil'kov [9]; they have stressed that in order to obtain a reliable extrapolation it is necessary to have experimental data in kinematical condition as close as possible to the point $t = 0$, being $t$ the momentum transferred between the final neutron and the initial proton.

A measurement performed at 1.5 GeV photon energy, using polarized photons, would fulfill the experimental requirements of Ref. [9], also providing higher sensitivity to the pion polarizability contributions through the polarization structure functions.

A fundamental issue of the experimental set-up is the capability of discriminating the reaction of interest from the background events, like those coming from asymmetric decay of $\pi^0$ in the $\pi^+\pi^0 n$ reaction channel.

In principle all these requirements are fulfilled by the Graal facility: the scattered photons may be detected in the BGO ball for laboratory angles between 25° and 155°; low energy neutrons my be detected in the forward direction using the plastic wall and they may be identified using the TOF information; finally the pions may be detected at all angles by the wire chambers with good angular resolution. All these experimental information should allow a complete reconstruction of the interesting events in selected kinematical conditions, with expected good background rejection.

The Graal facility set-up is therefore a promising tool to perform the first experiment with polarized photons on radiative pion photoproduction in order to extract information on the pion polarizability.

Thomas Walcher has reviewed, from his general talk on the experimental activity at MAMI in Mainz, the measurement of the pion polarizability still by means of radiative pion photoproduction.

Walcher has shown some kinematical conditions suitable for the measurement

of the charged pion polarizability. The sensitivity in the Chew-Low extrapolation at the pion pole has been stressed [9]. For instance a variation of $\alpha_\pi$ from 0 up to 7 is equivalent to a 20 % variation in the extrapolated amplitude. Hence the extrapolation has to be done at the 2 % level , if $\alpha_\pi$ has to be measured at a 10 % level. This sensitivity depends, of course, on the minimum momentum transfer $t_{min}$ achieved. For instance for a 700 MeV incident photon, a 152 MeV final photon, a neutron in the angular range $12^0$-$32^0$ and in the energy range 400 - 350 MeV, it is $t$= 0.31, in pion mass squared units. Nucleon, pion and $\Delta$(1236) pole diagrams have been evaluated and the extrapolation seems feasible.

Conversely the measurement of the neutral pion polarizability is not realistic at the MAMI energies. The situation may be improved at higher incident photon energies and a polarized beam would be very welcome, just like the Graal facility! By the way M. Moinester has stressed that the maximum energy meaningful for extracting the pion polarizability is about 2 GeV, corresponding to the $\rho$ mass in the photon-pion c.m. system.

The MAMI detector consists of a MWPC system to detect the charged pion, close to the incident photon beam, a segmented $BaF_2$ to detect the scattered photon and a system of scintillators for detecting the neutron and providing the neutron time of flight.The expected yield of radiative pion photoproduction is $\simeq$ 2000 events/day. The background due to double photoproduction, simulating radiative pion photoproduction, is $\simeq$ 160 events/day.


## 2.2   Primakoff effect

The contribution of Murray Moinester has concerned the measurement of $\pi$ $(K)$ and $\Sigma$ polarizabilities via $\pi$ $(K)$ and $\Sigma$ high energy beams at Fermilab by E781 [10].

This experimental activity has been reported already in detail by M. Moinester in his contribution to the workshop. Therefore only the main topics are emphasized here, first of all the relationship between polarizability and radiative transitions.

The $A_1$ radiative width is a good illustration of this statement. It has been demostrated [11] according to the current algebra the main contribution to the pion electric polarizability comes from the exchange of the $A_1$. Xiong, Shuryak and Brown [12] have shown that a radiative width $\Gamma(A_1 \rightarrow \pi\gamma)$= 1.4 MeV is needed to get the current algebra value, on the contrary the experimental value is $\Gamma(A_1 \rightarrow \pi\gamma)$= 0.64 $\pm$ 0.25 MeV [13]. More data from E781 are welcome to settle this relevant problem.

By the way the unexpected correlation between the reaction $\pi\rho \to \pi\gamma$ and the photon flux in a quark-gluon plasma has been pointed out.

Experimental results from the previous FNAL experiment E272 have been shown to demonstrate the experimental feasibility of the Primakoff effect, even if E272 did not get enough statistics to measure $\alpha_\pi$. The main experimental problem in measuring the Primakoff effect by the new FNAL experiment, E781, concerns a suitable fast trigger. It is not implemented at the moment, taking into account the high rate and the difficulties for using any signal from the scattered photon detector, which is 50 meters downstream the target.

From a theoretical point of view an open question remains the fair disagreement between the charged $\rho$ radiative width, as obtained via the Primakoff effect, and the neutral $\rho$ radiative width.

Finally M. Moinester has stressed the role of radiative transitions in the case of a $\Sigma$ beam, which should also be available in E781. The radiative transitions to the Sigma*(1385) provide a measure of the s-quark magnetic moment of the Sigma[14]. Positive and negative $\Sigma$ are expected to have very different polarizabilities. In particular the $\Sigma^-$ magnetic moment should be negligible, both taking into account the 3 quarks have the same charge and according to the U-spin symmetry. Furthermore it has been shown that the event rate for measuring $\Sigma$ radiative transitions by a 600 GeV $\Sigma$ beam is higher than the expected rate for measuring the pion Primakoff effect by E781.

S. Kananov has reported about radiative corrections in the scattering of pions by nuclei at high energies [15]. It has been shown that radiative corrections can simulate a variation of the magnetic polarizability $\beta_\pi \simeq$ -0.2 from $\beta_\pi \simeq$ -5, with plausible cuts for the outgoing photon.

## 2.3  $\gamma\gamma \to \pi\pi$ at threshold

Another way to get the pion polarizability is by means of the measurement of $\gamma\gamma \to \pi\pi$ at threshold, performed at the new Frascati $\Phi$-factory DA$\Phi$NE [16].

A presentation of this new experimental facility has been done already in the workshop and no further discussion on experimental details has been done in this working group.

In summary the new Frascati $e^+e^-$ storage ring DAΦNE is supposed to deliver a luminosity $\simeq 5 \cdot 10^{32}$ cm$^2$ $s^{-1}$ at the Φ mass, with the possibility to increase the total energy up to 1.5 GeV. Two detectors are under construction: an all purposes detector, KLOE [17] mainly dedicated to CP violation in $K$ decay, and FINUDA [18] mainly dedicated to hypernuclei physics.

KLOE is expected [19] to detect $10^3 \div 10^4$ times the events collected at present in $e^+e^- \to e^+e^-\pi^+\pi^-$ and $e^+e^- \to e^+e^-\pi^0\pi^0$ at threshold. Unfortunately at the Φ energy the decay $\Phi \to K_S K_L$ is an overwhelming background (in $\sim$ 15 % of the events $K_L$ are not detected) and tagging the outgoing $e^+e^-$ is needed. Two kind of tagging system for the outgoing leptons are foreseen. First of all there are two different rings for electrons and positrons and the splitter magnet after the interaction region is a suitable magnetic analyzer for the outgoing $e^+e^-$, mostly forward emitted. Furthermore $e^+e^-$ emitted at larger angles are, in part, detected by the central tracking detector in KLOE. The $e^+e^-$ angular distribution depends on $m_e/E_e$ : therefore there are more events at large angles in DAΦNE respect to the high energy $e^+e^-$ storage rings.

By the way correlations in the azimuthal angles between the pions and the outgoing leptons could be performed [20], increasing the possibility to disentangle the D wave contribution. Otherwise $\gamma\gamma$ interactions near threshold are supposed to provide mainly the $\pi\pi$ S wave, which provides only $\alpha_\pi - \beta_\pi$.

The overall double tagging efficiency is $\sim$ 15 % [19]. The background from beam-beam bremmstrhalung is still under study for evaluating the single tagging efficiency.

In $\gamma\gamma$ interactions complications related to any nuclear target are avoided, but it has been demonstrated in the following theoretical discussion that the extrapolation to the pion pole is much more difficult. Therefore the conclusion has been achieved that $\gamma\gamma$ interactions are not the best way to get the pion polarizability. Nevertheless $\gamma\gamma$ interactions near threshold remain a very clean test of any theoretical description of strong interactions at low energies.

Another possibility pointed out for measuring neutral and charged $\alpha_\pi$ in $e^+e^-$ is by means of $e^+e^- \to \pi\pi\gamma$ increased by the interference both with $\omega \to \pi^0$ $\rho^0 \to \pi_0\pi^0\gamma$ and, for the charged one, also with radiative $\rho$ production.

# 3  Theoretical issues in polarizability experiments

We begin with the process $\gamma\gamma \to \pi^0\pi^0$. In this case the Born amplitude vanishes and the one-loop corrections in Chiral Perturbation Theory (CHPT)[21]-[25] are finite [26, 27]. The corresponding cross section is independent of the free parameters of the chiral lagrangian and does not agree with the experimental measurements at Crystal Ball [28], as well as with calculations based on dispersion relations [29]-[35], even at low-energy. The low-energy amplitude recently calculated to two-loops in CHPT [36] agrees with the Crystal Ball data and compares very well with the results of a dispersive calculation by Donoghue and Holstein [32].

The value of the low-energy constants can be obtained in several ways, e.g. by resonance exchange. The resonance saturation method provides empirical values for the scale-dependent renormalized constants of CHPT [22]-[23]

$$L_i^r(\mu) = L_i^r(\mu_0) - \frac{\Gamma_i}{16\pi^2} ln\frac{\mu}{\mu_0} \quad , \ i = 1,..,10 \tag{3.1}$$

with a scale $\mu$ in the range $0.5 \ GeV - 1 \ GeV$ and a set of constants $\Gamma_i$ defined in [22]-[23]. This method has been used in Ref. [36] to pin down the couplings in the $\gamma\gamma \to \pi^0\pi^0$ amplitude to order $p^6$. In his talk J. Kambor discussed how to determine these couplings from sum rules, exploiting the low- and high-energy behaviour.

Let us consider the vector-vector two-point function

$$i \int d^4x e^{iqx} < 0|T(V_\mu^a(x)V_\nu^b(0)|0 >= \delta^{ab}(q_\mu q_\nu - g_{\mu\nu}q^2)\Pi^a(q^2). \tag{3.2}$$

Following [21]-[23] and [32] we write a dispersion relation for $\Pi^a(q^2)$

$$\Pi^a(q^2) = \frac{1}{\pi} \int ds \frac{Im\Pi^a(s)}{s - q^2 - i\epsilon} \ + \ subtractions \ , \tag{3.3}$$

in order to make contact with the high-energy behaviour of the theory desumed from the perturbative $QCD$ sum rules

$$\rho_V^a(s) = \frac{1}{\pi} Im\Pi^a(s) \ ,$$
$$lim_{s\to\infty}\rho_V^a(s) = \frac{1}{8\pi^2} + O(\frac{1}{s}) \ , \ a = 3, 8 \tag{3.4}$$

showing that the spectral function $\rho_V^a(s)$ at high energy goes like a constant plus higher-order terms that are suppressed at least as $\frac{1}{s}$. Hence, the difference between two spectral functions goes like $s$ at large $s$, and the integral

$$\int ds \frac{\rho_V^3 - \rho_V^8}{s} \tag{3.5}$$

converges. Also the once-subtracted dispersion relation for $\rho_V^3(s)$ converges

$$\int ds \frac{\rho_V^3}{s^2} \ . \tag{3.6}$$

From the dispersion relation

$$\Pi^3(q^2) - \Pi^8(q^2) = \frac{1}{\pi} \int ds \frac{Im\Pi^3(s) - Im\Pi^8(s)}{s - q^2 - i\epsilon} \tag{3.7}$$

the sum rule is readily obtained

$$\Pi^3(0) - \Pi^8(0) = \int ds \frac{\rho_V^3 - \rho_V^8}{s} \ . \tag{3.8}$$

Taking the $q^2$ derivative of the dispersion relations (3.7) and (3.3) yields the sum rules

$$\frac{d}{dq^2}(\Pi^3(0) - \Pi^8(0)) = \int ds \frac{\rho_V^3 - \rho_V^8}{s^2} \tag{3.9}$$

and, respectively,

$$\frac{d}{dq^2}\Pi^3(0) = \int ds \frac{\rho_V^3}{s^2} \ . \tag{3.10}$$

Notice that if one takes too many $q^2$-derivatives, then the integrals become dominated by the threshold region.

As for the low-energy behaviour, J. Kambor showed how to use CHPT to calculate $\Pi^a(q^2)$ for small $q^2$ values. This calculation is carried out in $SU(3) \times SU(3)$ to the two-loop order, and the result depends on the $O(p^4)$ and $O(p^6)$ low-energy constants $L_i$ and, respectively, $d_j$ [37]. The integral on the r.h.s. of Eq. (3.8) can be evaluated from the $e^+e^-$ data. In the narrow width approximation one gets the following estimate [38]:

$$\int ds \frac{\rho_V^3 - \rho_V^8}{s} = \frac{3}{4\pi\alpha^2}\left(\frac{\Gamma_{\rho \to e^+e^-}}{M_\rho} - 3\frac{\Gamma_{\omega \to e^+e^-}}{M_\omega} - 3\frac{\Gamma_{\phi \to e^+e^-}}{M_\phi}\right) = (11.1 \pm 2.0) \cdot 10^{-3} \ . \tag{3.11}$$

Thus, the integration region becomes divided into three pieces, i.e. $4M_\pi^2 \leq s \leq \Lambda_1$, $\Lambda_1 \leq s \leq \Lambda_2$, and $s \geq \Lambda_2$. Here we denote by $\Lambda_{1,2}$ two cutoff values of about 0.4 $GeV$ and 2 $GeV$, respectively, and $M_\pi$ is the pion mass. In the first region the shape of the spectral function is obtained from the two-loop CHPT calculation, i.e. $\rho_V = \rho_V^{1-loop} + \rho_V^{2-loop}$. In the second (third) region the $e^+e^-$ data (the perturbative $QCD$ calculation) can be used to obtain $\rho_V(s)$. The result of this calculation (once it is completed) yields a scale independent determination of $d_3$ contributing to $\gamma\gamma \to \pi^0\pi^0$. J. Kambor expects an accuracy for this estimate between 10 and 20 percent.

9

This would be more precise than the estimate carried out in [38] using a similar procedure (excluding, however, the two-loop contribution), within the framework of the Generalized Chiral Perturbation Theory (GCHPT)

$$d_3 = (9.4 \pm 4.7) \cdot 10^{-6} \; . \tag{3.12}$$

There are two more sum rules to consider. In particular Eq. (3.9) is effectively a sum rule for $L_9$ (the $d_i$ contributions to $\Pi^3$ and $\Pi^8$ drop out of the sum rule), whereas Eq. (3.10) is a sum rule for $d_5$, $d_6$ contributing to $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\pi^\pm \to \gamma\pi^\pm$. A similar treatment can be applied to the sum rule for the axial-axial two-point function. This gives an estimate of the low-energy constants $L_{1,2,3}$ contributing to $\pi\pi$-scattering.

The GCHPT approach is described in [39] (see also M. Knecht's talk in the $\pi\pi$-scattering Working Group Section of these Proceedings). Within this approach the cross section for $\gamma\gamma \to \pi^0\pi^0$ and the pion polarizabilities have been calculated up to and including $O(p^5)$ [38]. M. Knecht discussed the result of this calculation. He showed that the cross section depends on the quark mass ratio $r = \frac{2m_s}{m_u+m_d}$ and is consistent with the data from Crystal Ball [28], provided the value of the ratio is at least a factor of two or three smaller than its standard value in CHPT [38]. M. Knecht showed that a low-energy theorem analogous to the one outlined above, but without taking into account the 2-loop contribution, yields, through the evaluation of the dispersive integral via resonance saturation, the following value for the $O(p^5)$ constant $c$ defined in [38]:

$$c = -\frac{1}{r-1}(4.6 \pm 2.3) \cdot 10^{-3} \; . \tag{3.13}$$

In the standard CHPT case, i.e. for

$$r = 2\frac{M_K^2}{M_\pi^2} - 1 = 25.9 \; , \tag{3.14}$$

the expression for $c$ given in Eq. (3.13) corresponds to the value reported in Eq. (3.12). This is to be compared with the value calculated from Appendix D of Ref. [36], using resonance exchange

$$d_3 = \pm 3.9 \cdot 10^{-6} \; . \tag{3.15}$$

M. Pennington reviewed the dispersion relation treatment of the $\gamma\gamma \to \pi^0\pi^0$ amplitude that represents the data quite well [30], [31]. He showed how to calculate

10

the cross section from first principles, using a relativistic and causal description based on the unitarity of the scattering matrix. The prediction for low-energy $\gamma\gamma \to \pi^0\pi^0$ and $\gamma\gamma \to \pi^+\pi^-$ is based on the present knowledge of the $\pi\pi$ phases and $PCAC$. M. Pennington argued that precision measurement of $\gamma\gamma \to \pi^0\pi^0, \pi^+\pi^-$ at the Frascati $\Phi$-factory DA$\Phi$NE will restrict further the $\pi\pi$ phases and determine where the chiral zero appears on-shell.

The electric and magnetic polarizabilities enter the low-energy limit of the coupling with the photon in the Compton amplitude for any composite system. The dynamics of hadronic systems can be probed by measuring the hadron polarizabilities [40]. In particular, the pion Compton scattering can be investigated in this connection. The charged pion Compton amplitude

$$\gamma(q_1)\pi^+(p_1) \to \gamma(q_2)\pi^+(p_2) \ , \tag{3.16}$$

admits an expansion near threshold

$$T^C = 2\left[\vec{\epsilon}_1 \cdot \vec{\epsilon}_2^\star \left(\frac{\alpha}{M_\pi} - \bar{\alpha}_\pi \omega_1\omega_2\right) - \bar{\beta}_\pi (\vec{q}_1 \times \vec{\epsilon}_1) \cdot (\vec{q}_2 \times \vec{\epsilon}_2^\star) + \cdots\right] \tag{3.17}$$

with $q_i^\mu = (\omega_i, \vec{q}_i)$. Below we denote

$$\begin{aligned}
(\alpha \pm \beta)^C &= \bar{\alpha}_\pi \pm \bar{\beta}_\pi \ , \\
(\alpha \pm \beta)^N &= \bar{\alpha}_{\pi^0} \pm \bar{\beta}_{\pi^0} \ , 
\end{aligned} \tag{3.18}$$

for charged and neutral pions, respectively.

The charged pion polarizabilities have been determined in an experiment on the radiative pion-nucleus scattering $\pi^- A \to \pi^- \gamma A$ [41] and in the pion photoproduction process $\gamma p \to \gamma \pi^+ n$ [42]. Assuming the constraint $(\alpha + \beta)^C = 0$ the two experiments yield[1]

$$(\alpha - \beta)^C = \begin{cases} 13.6 \pm 2.8 & [41] \\ 40 \pm 24 & [42] \ . \end{cases} \tag{3.19}$$

Relaxing the constraint $(\alpha + \beta)^C = 0$, one obtains from the Serpukhov data

$$\begin{aligned}
(\alpha + \beta)^C &= 1.4 \pm 3.1(\text{stat.}) \pm 2.5(\text{sys.}) \ [43] \ , \\
(\alpha - \beta)^C &= 15.6 \pm 6.4(\text{stat.}) \pm 4.4(\text{sys.}) \ [43] \ .
\end{aligned} \tag{3.20}$$

---

[1]Throughout the following, we express the values of the polarizabilities in units of $10^{-4}fm^3$

At one-loop in CHPT one has [44, 11, 45]

$$\bar{\alpha}_{\pi^0} = -\bar{\beta}_{\pi^0} = -\frac{\alpha}{96\pi^2 M_\pi F^2} = -0.50 \ . \tag{3.21}$$

At order $O(p^6)$ it was calculated in Ref. [36]

$$\begin{aligned}
\bar{\alpha}_{\pi^0} &= -0.35 \pm 0.10 \ , \\
\bar{\beta}_{\pi^0} &= 1.50 \pm 0.20.
\end{aligned} \tag{3.22}$$

The low-energy $\gamma\gamma \to \pi^+\pi^-$ data [46] have been used in Ref. [45] to obtain information on $\bar{\alpha}_\pi$ and $\bar{\beta}_\pi$. The result in [45] yields the numerical value for the leading-order $\bar{\alpha}_\pi = 2.7 \pm 0.4$, plus systematic uncertainties due to the $O(p^6)$ corrections. The latter are not yet available. A part of the corrections to the charged pion polarizabilities beyond the one-loop order has been obtained in Refs. [47, 48] including the meson resonance contribution.

M. Knecht analyzed the $O(p^5)$ calculation from Ref. [38] of the $\pi^0$ polarizabilities in GCHPT. The result $(\alpha + \beta)^N = 0$ remains valid to this order, whereas the remaining combination depends on the quark mass ratio $r$. Hence this combination can have positive values for $r$ much less than its standard CHPT value (3.14), e.g. $(\alpha - \beta)^N = 1.04 \pm 0.60$ for $r = 10$ [38]. For comparison we recall the standard CHPT prediction for both combinations to the $O(p^6)$ order [36]

$$\begin{aligned}
(\alpha + \beta)^N &= 1.15 \pm 0.30 \ , \\
(\alpha - \beta)^N &= -1.90 \pm 0.20.
\end{aligned} \tag{3.23}$$

The reason for the sign difference of $(\alpha - \beta)^N$ in GCHPT with respect to the standard CHPT value has been traced back by M. Knecht to a dominance by the positive $O(p^5)$ contribution over the strongly suppressed negative $O(p^4)$ contribution in GCHPT. This suppression is related to a shift in the position of the chiral zero, as $r$ becomes much smaller than the standard CHPT value (3.14) [38].

Starting from the unitarized $S$-wave amplitudes for neutral pions, M. Pennington displayed a proportionality relation between $(\alpha - \beta)^N$ and the position of the chiral zero $s_N$, showing that the former assumes values between -0.6 and -2.7 as the latter runs from $\frac{1}{2}M_\pi^2$ and $2M_\pi^2$. He also discussed the validity of the errors quoted in a recent estimate of $(\alpha + \beta)^{C,N}$ by Kaloshin and collaborators [49]. Here the polarizabilities appear as adjustable parameters in the unitarized $D$-wave amplitudes, hence the values of $(\alpha + \beta)^{C,N}$ can be determined from the data with the result [49]

$$\begin{aligned}
(\alpha + \beta)^C &= 0.22 \pm 0.06 \ [46] \ , \\
(\alpha + \beta)^N &= 1.00 \pm 0.05 \ [28] \ .
\end{aligned} \tag{3.24}$$

M. Pennington, arguing on the partial wave analysis of the data that shows large uncertainties even at the $f_2(1270)$ mass, concluded that the errors quoted in (3.24) for $(\alpha + \beta)^N$ are unbelievably small. His final conclusion, that one must measure $\gamma\pi \to \gamma\pi$ can be wholeheartedly shared, in view of future measurements of the pion polarizabilities. In this respect, it is very important to devise fully reliable methods that allow to extract the pion Compton scattering amplitude from the measurement of the radiative pion photoproduction, as discussed by the author.

# References

[1] D. Babusci, L. Casano, A. D'Angelo, P. Picozza, C. Schaerf and B. Girolami, Internal report LNF-90/060(P) - Nuovo Cimento 103 A (1990) 1555.

[2] D. Babusci, L. Casano, A. D'Angelo, B. Girolami, D. Moricciani, P. Picozza, C. Schaerf, Proc. Workshop "Future of Nuclear Physics in Europe with Polarized Electrons and Photons" Orsay July 4-6 (1990) 149.

[3] J.P. Bocquet, M. Capogni, L. Casano, A. D'Angelo, F. Ghio, B. Girolami, D. Moricciani, C. Perrin, P. Picozza, C. Schaerf, Book of abstr. PANIC XIII Perugia (1993) 835.

[4] M. Anghinolfi, V. Bellini, G. Berrier, N. Bianchi, J. P. Bocquet, H. Bougnet, P. Calvat, M. Capogni, L. Casano, P. Corvisiero, A. D'Angelo, E. De Sanctis, J.P. Didelez, Ch. Djalali, M.A. Duval, A. Ebolese, A. Elayi, A. Fantoni, R. Frascaria, G. Gervino, F. Ghio, B. Girolami, E. Hourani, P. Levi Sandri, L. Mazzaschi, V. Mokeev, D. Moricciani, M. Morlet, V. Muccifora, C. Perrin, P. Picozza, E. Polli, D. Rebreyend, A.R. Reolon, G. Ricco, M. Rigney, M. Ripani, R. Rosier, P. Rossi, Th. Russew, M. Sanzone, C. Schaerf, M. Taiuti, J. Van de Wiele, A. Zucchiatti, Book of abstr. PANIC XIII Perugia (1993) 850.

[5] The BGO Collaboration, Internal report LNF-90/084(1990).

[6] L. Mazzaschi, P. Levi Sandri, A. Zucchiatti, M. Anghinolfi, D. Babusci, N. Bianchi, P. Corvisiero, A. D'Angelo, E. De Sanctis, G. Gervino, B. Girolami, V. Lucherini, V. Mokeev, V. Muccifora, P. Picozza, E. Polli, A.R. Reolon, G. Ricco, M. Ripani, P. Rossi, M. Sanzone, C. Schaerf, M. Taiuti, Nucl. Instr. and Meth. A305 (1991) 391-394.

[7] A. Zucchiatti, M. Castoldi, G. Gervino, F. Terzi, M. Anghinolfi, N. Bianchi, L. Casano, P. Corvisiero, A. D'Angelo, E. De Sanctis, B. Girolami, P. Levi Sandri, V. Lucherini, L. Mazzaschi, V.I. Mokeev, V. Muccifora, P. Picozza, E. Polli, A.R. Reolon, G. Ricco, M. Ripani, P. Rossi, M. Sanzone, C. Schaerf, M. Taiuti, Nucl. Instr. and Meth. A317 (1992) 492.

[8] A. Zucchiatti, P. Levi Sandri, C. Schaerf, A.A. Cowley, J.V. Pilcher, L. Mazzaschi, M. Anghinolfi, N. Bianchi, L. Casano, P. Corvisiero, A. D'Angelo, E. De Sanctis, B. Girolami, V. Lucherini, V.I. Mokeev, V. Muccifora, P. Picozza, E.

Polli, A.R. Reolon, G. Ricco, M. Ripani, P. Rossi, M. Sanzone, M. Taiuti, Nucl. Instr. and Meth. A321 (1992) 219.

[9] D. Drechsel and L.V. Fil'kov, Internal Report MKPH-T-94-4.

[10] M. Moinester, Proceedings of the Conference on the Intersection between Particle and Nuclear Physics, Tucson(1991) 553, Ed.W.Van Oers.

[11] B.R. Holstein, Comments Nucl. Part. Phys. 19 (1990) 221.

[12] L. Xiong, E. Shuryak, G. Brown, Phys. Rev. 46D (1992) 3798.

[13] M. Zieliski et al., Phys. Rev. Lett. 52 (1984) 1195.

[14] H.J. Lipkin, M.A. Moinester, Phys. Lett. B287 (1992) 179.

[15] A. A. Akhundov, S. Gerzon, S. Kananov, M. A. Moinester, TAUP-2183-94 (1994).

[16] M. E. Biagini, PANIC Proceedings, Perugia (1993) 763.

[17] KLOE Collaboration Lab. Naz. di Frascati LNF-92/019 (1993), KLOE Collaboration (addendum), LNF-94/028 (1994).

[18] FINUDA Collaboration Lab. Naz. di Frascati LNF-93/021 (1993).

[19] G. Alexander et al., Il Nuovo Cimento 107A (1994) 837.

[20] S. Ong, P. Kessler and A. Courau, Mod. Phys. Lett. A 4 (1989) 909.

[21] S. Weinberg, Physica 96A (1979) 327.

[22] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) 158 (1984) 142.

[23] J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 465.

[24] H. Leutwyler, Bern University preprint BUTP-93/24 (hep-ph/9311274).

[25] For recent reviews on CHPT see e.g.
H. Leutwyler, in: Proc. XXVI Int. Conf. on High Energy Physics, Dallas, 1992, edited by J.R. Sanford, AIP Conf. Proc. No. 272 (AIP, New York, 1993) p. 185;
U.G. Meißner, Rep. Prog. Phys. 56 (1993) 903;
A. Pich, Lectures given at the V Mexican School of Particles and Fields, Guanajuato, México, December 1992, preprint CERN-Th.6978/93 (hep-ph/9308351);

G. Ecker, Lectures given at the 6th Indian–Summer School on Intermediate Energy Physics Interaction in Hadronic Systems Prague, August 25 - 31, 1993, to appear in the Proceedings (Czech. J. Phys.), preprint UWThPh -1993-31 (hep-ph/9309268).

[26] J. Bijnens and F. Cornet, Nucl. Phys. B296 (1988) 557.

[27] J.F. Donoghue, B.R. Holstein and Y.C. Lin, Phys. Rev. D37 (1988) 2423.

[28] The Crystal Ball Collaboration (H. Marsiske et al.), Phys. Rev. D41 (1990) 3324.

[29] R.L. Goble, R. Rosenfeld and J.L. Rosner, Phys. Rev. D39 (1989) 3264. The literature on earlier work may be traced from this reference.

[30] D. Morgan and M.R. Pennington, Phys. Lett. B272 (1991) 134.

[31] M.R. Pennington, in The DAΦNE Physics Handbook, edited by L. Maiani, G. Pancheri and N. Paver (INFN, Frascati, 1992), p. 379.

[32] J.F. Donoghue and B.R. Holstein, Phys. Rev. D48 (1993) 137.

[33] A. Dobado and J.R. Peláez, Z. Phys. C57 (1993) 501.

[34] T.N. Truong, Phys. Lett. B313 (1993) 221.

[35] A.E. Kaloshin and V.V. Serebryakov, Irkutsk preprint ISU-IAP.Th93-03 (hep-ph/9306224).

[36] S. Bellucci, J. Gasser and M.E. Sainio, Nucl. Phys. B423 (1994) 80.

[37] E. Golowich and J. Kambor, in preparation.

[38] M. Knecht, B. Moussallam and J. Stern, Orsay preprint IPNO/TH 94-08, to appear in Nuc. Phys. B.

[39] N.H. Fuchs, H. Sazdjian and J. Stern, Phys. Lett. B269 (1991) 183.

[40] V.A. Petrunkin, Sov. J. Part. Nucl. 12 (1981) 278;
J.L. Friar, in: Proc. of the Workshop on Electron-Nucleus Scattering, Marciana Marina, 7-15 June 1988, eds. A. Fabricini et al., (World Scientific, Singapore, 1989) p. 3;
M.A. Moinester, in: Proc. 4th Conf. on the Intersections Between Particle and Nuclear Physics, Tucson, Arizona, May 24-29, 1991, AIP Conf. Proc. No. 243, ed. W.T.H. Van Oers (AIP, New York, 1992), p. 553.

[41] Yu.M. Antipov et al., Phys. Lett. 121B (1983) 445; Z. Phys. C24 (1984) 39.

[42] T.A. Aibergenov et al., Czech. J. Phys. B36 (1986) 948.

[43] Yu.M. Antipov et al., Z. Phys. C26 (1985) 495.

[44] J.F. Donoghue and B.R. Holstein, Phys. Rev. D40 (1989) 2378.

[45] D. Babusci et al., in The DAΦNE Physics Handbook, edited by L. Maiani, G. Pancheri and N. Paver (INFN, Frascati, 1992), p. 383;
D. Babusci et al., Phys. Lett. B277 (1992) 158.

[46] The Mark II Collaboration (J. Boyer et al.), Phys. Rev. D42 (1990) 1350.

[47] A.E. Kaloshin and V.V. Serebryakov, Z. Phys. C32 (1986) 279.

[48] D. Babusci, S. Bellucci, G. Giordano and G. Matone, Phys. Lett. B314 (1993) 112.

[49] A.E. Kaloshin, V.M. Persikov and V.V. Serebryakov, "First estimates of the $(\alpha + \beta)^{\pi}$ from two-photon experiments", Irkutsk preprint ISU-IAP.Th94-01.